A NON–SUPERSYMMETRIC DYONIC EXTREME REISSN BLACK HOLE

Ramzi R. Khuri[a,b] [1] **and Tomás Ortín**[a] [2]

[a] *C.E.R.N. Theory Division*
CH–1211, Genève 23, Switzerland

[b] *McGill University*
Physics Department
Montreal, PQ, H3A 2T8 Canada

Abstract

We present a dyonic embedding of the extreme Reissner-Nordström black hole in $N = 4$ and $N = 8$ supergravity that breaks all

CERN-TH/95–347
McGill/96–01

[1] E-mail address: khuri@nxth04.cern.ch
[2] E-mail address: tomas@surya20.cern.ch

It has recently been shown [1] that all possible embeddings of purely electric or purely magnetic four-dimensional extreme dilaton black holes in $N = 8$ supergravity must be related by duality symmetries of the $N = 8$ supergravity theory ("U duality" [2]) and therefore must have the same number of unbroken supersymmetries in $N = 8$. As explained in Ref. [1], the same statement is not true in the context of the $N = 4$ theory and other truncations of the $N = 8$ theory with fewer supersymmetries because some of the U duality transformations that relate the embeddings (which are also strictly $N = 4$ embeddings) are *not* duality symmetries of the $N = 4$ theory, and change the number of unbroken $N = 4$ supersymmetries.

A well-known example is that of the $a = 1$ extreme dilaton black hole solution of the action [3]

$$S^{(a)} = \tfrac{1}{2}\int d^4x\ \sqrt{|\tilde{g}|}\left[-\tilde{R} - 2(\partial\varphi)^2 + \tfrac{1}{2}e^{-2a\varphi}F^2\right]. \tag{1}$$

When Garfinkle, Horowitz and Strominger rediscovered this solution in the context of string theory [4], they made the claim that, in spite of the fact that the solution saturates a Bogomol'nyi-type bound, it is not supersymmetric because the photino supersymmetry transformation law was always different from zero for any supersymmetry parameter. In other words, when the vector field appearing in the action considered by the authors was taken to be a matter vector field belonging to a vector supermultiplet, the supersymmetry variation of the spinor present in this supermultiplet could not be made to vanish, and the solution was not supersymmetric.

Shortly afterwards [5] it was discovered that, when the vector field is considered as one of the six vector fields present in the $N = 4, D = 4$ supergravity multiplet, there exists a choice of local supersymmetry parameter such that the supersymmetry transformation laws of the four gravitini and dilatini of this supermultiplet vanish; *i.e.* there exists a Killing spinor such that the supersymmetry variation vanishes and the extreme dilaton black hole is supersymmetric.

The choice of considering the vector field as a matter or as a supergravity vector field corresponds to a different choice of *embedding* of the same four-dimensional solution in the ten-dimensional $N = 1$ supergravity theory which corresponds to the $N = 4$ supergravity theory plus six vector supermultiplets in four dimensions upon dimensional reduction [6].

This seemingly paradoxical situation in which the same solution is or is not supersymmetric depending on the embedding in $N = 1, D = 10$ supergravity chosen is resolved by further embedding into any of the $N = 2, D = 10$ theories [1]. The six matter vector fields of the $N = 4, D = 4$ theory become supergravity vector fields of the $N = 8, D =$
reduction of $N = 2, D = 10$ supergr
proposed in Refs. [4, 5] have the same
ken (namely two).

The reason for this result (and a
that occur in $N = 8, D = 4$ supergr
of the same solution are related by
the $N = 8, D = 4$ supersymmetries.
symmetries of the $N = 4, D = 4$ theor
the U duality symmetry takes us fr
$N = 4, D = 4$ solution, the number o
is generally not preserved.

In this letter we present a new em
(*i.e.* the extreme Reissner-Nordström
supersymmetries in $N = 8, D = 4$
embeddings have one $N = 8, D = 4$
cannot be related to the known emb
also unique because it is the only en
dilaton black hole known (with just o

Since the embedding is determined
fields with the fields occurring in th
$N = 8, D = 4$ action. In fact, thi
$N = 4, D = 4$ (plus six vector superm
theory, whose action is [7]

$$S = \tfrac{1}{2}\int d^4x\ \sqrt{|g|}$$

$$+\tfrac{1}{4}\left[\partial G_{mn}\partial G^{\cdots}\right.$$

$$-\tfrac{1}{4}\left[G_{mn}F^{(1)\cdots}\right.$$

It can be easily checked that the eq
those of the theory (1) with $a = 0$ wi

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}\,, \qquad F^{(1)1}{}_{\mu\nu} =$$

where F is either purely electric or
Then, any solution with constant sca

field of the Einstein-Maxwell-scalar theory can be considered as a solution of the above $N=4, D=4$ theory, and, henceforth, of the $N=8, D=4$ theory. In particular, one can take the purely electric ERN solution

$$\left\{\begin{array}{rcl} d\tilde{s}^2 & = & V^{-2}dt^2 - V^{+2}d\vec{x}^2\,, \\ \\ F_{t\underline{i}} & = & -\sqrt{2}\; n\; \partial_{\underline{i}} V^{-1}\,, \end{array}\right. \tag{4}$$

where $V(\vec{x})$ is a harmonic function in three-dimensional Euclidean space $\partial_{\underline{i}}\partial_{\underline{i}}V = 0$

$$V(\vec{x}) = 1 + \sum_i \frac{M_i}{|\vec{x}-\vec{x}_i|}\,, \qquad M_i \geq 0 \;\; \forall i = 1,2,3\,, \tag{5}$$

and $n = \pm 1$ gives the sign of all the electric charges.

The advantage of the form (2) of the $N=4, D=4$ theory is that the relation between the four-dimensional and the ten-dimensional fields is simple and known [7, 1]. This allows us to rewrite the dyonic ERN solution (4) as a solution of $N=1, D=10$ supergravity (and, therefore, of the two $N=2, D=10$ theories):

$$\left\{\begin{array}{rcl} d\hat{s}^2 & = & V^{-2}dt^2 - V^2 d\vec{x}^2 - \left[dx^{\underline{4}} + \sqrt{2}n\;\left(V^{-1}dt \pm V_{\underline{i}}dx^{\underline{i}}\right)\right]^2 \\ \\ & & -dx^{\underline{I}}dx^{\underline{I}}\,, \qquad\qquad I = 5,\ldots,9\,. \\ \\ \hat{B} & = & \hat{\phi} = 0\,, \end{array}\right. \tag{6}$$

where the $V_{\underline{i}}$'s are functions satisfying

$$\partial_{[\underline{i}}V_{\underline{j}]} = \tfrac{1}{2}\epsilon_{ijk}\partial_{\underline{k}}V\,, \tag{7}$$

and whose (local) existence is ensured by the harmonicity of V.

This configuration is purely gravitational in ten dimensions. (One can directly check that it is a solution of ten-dimensional $N=1$ supergravity *i.e.* the metric above is Ricci-flat[3].) This implies that both torsionful spin connections $\hat{\Omega}^{(\pm)} = \hat{\omega} \mp \frac{3}{2}\hat{H}$ are equal to the (torsionless) spin connection $\hat{\omega}$ whose (tangent space) components are

[3] We thank H.H. Soleng for help with the *Mathematica* package CARTAN.

$\hat{\omega}_0{}^{0i}$

$\hat{\omega}_4{}^{0i} = \hat{\omega}_i{}^{04} = \hat{\omega}_0{}^{i4}$

$\hat{\omega}_4{}^{ij} = \hat{\omega}_j{}^{i4}$

$\hat{\omega}_k{}^{ij}$

The vanishing of the axion and d
$N=2, D=10$ supersymmetry rules

$\delta_\epsilon\hat{\psi}^{(\pm)}_{\hat{a}}$

where the superscript ${}^{(\pm)}$ of the spinc
and $\hat{\nabla}_{\hat{a}}$ is the covariant derivative ass
components can be read in Eqs. (8).

Now, if this configuration is goin
supersymmetry, the corresponding K
six compact coordinates $x^{\underline{4}},\ldots,x^{\underline{9}}$ nc

$\partial_t\hat{\epsilon}^{(\pm)} = \partial_{\underline{4}}\hat{\epsilon}^{(\pm)}$

and, substituting these conditions an
zero component of the Killing spinor

$\delta\hat{\psi}^{(\pm)}_0 = 0 \;\; \Rightarrow$

The matrix $\hat{\gamma}_0\hat{\gamma}_4$ squares to one, whi
and from this it follows that $\hat{\epsilon}^{(\pm)} =$
Had the coefficient on the r.h.s. bee
could be fulfilled for some spinors. T
to a continuous electric-magnetic dual
ERN black hole that transforms it into
of equal absolute value.

This result is in direct contrast to
magnetic ERN ($a=0$) black holes [1],
preserved in $N=8$ (but not in N
solution considered here is dyonic. C
supersymmetric embedding of the d
to the truncation from $N=8$ to N

theory, the purely electric or magnetic ERN black holes are supersymmetric and the electric-magnetic duality of this theory respects supersymmetry (see, for instance, the second lecture in Ref. [8]). The supersymmetric embedding in the $N = 8$ theory of the ERN dyonic black hole must have more than one non-vanishing vector field.

An intriguing possible explanation of the nonsupersymmetric nature of the dyonic black hole is the following: just as solutions which preserved different numbers of supersymmetries in $N = 4$ but the same number in $N = 8$ were argued to be unrelated by U duality transformations of $N = 4$ but connected by U duality transformations in $N = 8$, the question can be raised as to whether the present embedding is related to another supersymmetric embedding (for instance those of its purely electric or magnetic counterparts) by a duality transformation which is not a U duality transformation. That would lead us to speculate about the existence of a supergravity theory *larger* than $N = 8$ of which that duality would be a symmetry, just as C duality is a symmetry in the $N = 8$ theory but not in its $N = 4$ truncations.

Higher supergravity theories (higher than $N = 1, D = 11$) are usually considered "unphysical" because they must necessarily contain particles with helicities higher than two [9] and then consistency problems arise in the quantum field theory of these particles. As pointed out in Ref. [10], this could just be a prejudice reflecting the current status of local field theory. However, there are other serious problems in higher supergravity theories, such as the existence of more than one metric.

Still, even if this higher supergravity theory were physically unacceptable, one could still conceive that such an unphysical theory exists and is such that *it has a consistent physically acceptable truncation.* We have in mind the analogues of the truncations of the type II theories that render $N = 1, D = 10$ supergravity: one consistently truncates a set of fields (the R-R fields) and gets another supergravity theory. In the higher supergravity case, all the unwanted fields (higher helicities and further metrics, for instance) should disappear in this truncation. The result should be a known, physically acceptable, supergravity theory.

It is tempting to take this speculation a bit further and try to identify the truncated theory. A good candidate could be the type IIB theory, and a possible scenario could be the following: the higher supergravity theory could be non-chiral $N = 4, D = 10$, and there would be a chiral truncation in which all unphysical fields disappear and one chiral sector also disappears. Since the chirality of the type IIB theory is related to the self-or anti-self-duality of the five-form, and both chiralities would be present in this higher supergravity, the whole of the five form would also be present. Although it is highly speculative,

this scenario is particularly attractiv
could be obtained from higher dimen
be "explained" in a fashion analogou
higher supergravity theory could enc
have a truncation giving $N = 1, D$
Observe that type IIB supergravity ha
this would be consistent with its high

Perhaps there are other explanati
picture of extremal black holes [16], th
Furthermore, non-supersymmetric E
states and arising from a more genera
large classes of non-supersymmetric I
charge were found in Refs. [15]. Th
render an explanation of our results
that identical four-dimensional metri
are different and inequivalent under
properties, a proposition which we st

Acknowledgements

T.O. would like to thank M.M. Ferna
R.K. would like to thank the East Ca
hospitality. R.K. was supported by a

References

[1] R.R. Khuri and T. Ortín, *Supers*
ity, Report CERN-TH/95–348, N

[2] C.M. Hull and P.K. Towns
Nucl. Phys. **B438** (1995) 109.

[3] G.W. Gibbons, *Antigravitating*
$N = 4$ *Supergravity*, *Nucl. Phys.*
G.W. Gibbons and K. Maeda, *I*
mensional Theories with Dilaton

[4] D. Garfinkle, G. T. Horowitz an
String Theory, *Phys. Rev.* **D43**

[5] R. Kallosh, A. Linde, T. Ortín, A. Peet and A. Van Proeyen, *Supersymmetry as a Cosmic Censor*, *Phys. Rev.* **D46** (1992) 5278-5302.

[6] A. Chamseddine, *$N = 4$ Supergravity Coupled to $N = 4$ Matter*, *Nucl. Phys.* **B185**, (1981) 403.

[7] J. Maharana and J.H. Schwarz, *Non-Compact Symmetries in String Theory*, *Nucl. Phys.* **B390** (1993) 3.

[8] G.W. Gibbons, *Aspects of Supergravity Theories*, (three lectures) in: *Supersymmetry, Supergravity and Related Topics*, eds. F. del Aguila, J. de Azcárraga and L. Ibáñez, World Scientific, Singapore, 1985, page 147.

[9] W. Nahm, *Supersymmetries and their Representations*, *Nucl. Phys.* **B135** (1978) 149-166.

[10] J. Strathdee, *Extended Poincaré Supersymmetry*, *Int. J. Mod. Phys.* **A2** (1987) 273-300.

[11] J.H. Schwarz, *The Power of M Theory*, Report RU-95-68 and `hep-th/9510086`.
P. Hořava and E. Witten, *Heterotic and Type I String Dynamics from Eleven Dimensions*, Report IASSNS-HEP-95-86, PUPT-1571 and `hep-th/9510209`.

[12] E. Bergshoeff, B. Janssen and T. Ortín, *Solution-Generating Transformations and the String Effective Action*, `hep-th/9506156` (to be published in *Phys. Rev.* **D15**).

[13] M. Cvetič and D. Youm, *Dyonic BPS Saturated Black Holes of Heterotic String on a Six Torus*, Report UPR-0672-TA and `hep-th/9507090` (to be published in *Phys. Rev.* **D15**).
M. Cvetič and A. A. Tseytlin, *General Class of BPS Saturated Dyonic Black Holes as Exact Superstring Solutions*, Report IASSNS-HEP-95/79, Imperial/TP/95-96/4 and `hep-th/9510097` (to be published in *Phys. Lett.* **B**).

[14] M. Cvetič and D. Youm, *Singular BPS Saturated States and Enhanced Symmetries of Four-Dimensional N=4 Supersymmetric String Vacua*, *Phys. Lett.* **359B** (1995) 87-92.
M. Cvetič and D. Youm, *BPS Saturated and Non-Extreme States in Abelian Kaluza-Klein Theory and Effective N=4 Supersymmetric String Vacua*, talk given at the Strings'95 Conference, March 13-18 1995 in Los Angeles, CA (USA), Report UPR-675-T, NSF-ITP-95-74 and `hep-th/9508058`.

[15] H. Lü and C.N. Pope, *p-brane S*
CTP-TAMU-47/95, and `hep-th`
H. Lü and C.N. Pope, *Multi Sca*
52/95, and `hep-th/9512153`.
H. Lü and C.N. Pope, *An Approa*
Report CTP-TAMU-03/96, and

[16] J. Rahmfeld, *Extremal Black Ho*
51/95 and `hep-th/9512089`.